Pre-Calculus I

10.1 – Sequences and Summation Notation

Sequences

Definition of a Sequence

An **infinite sequence** $\{a_n\}$ is a function whose domain is the set of positive integers. The function values, or **terms**, of the sequence are represented by

$$a_1, a_2, a_3, a_4, \ldots, a_n, \ldots.$$

Sequences whose domains consist only of the first n positive integers are called **finite sequences**.

The graph of a sequence is a set of discrete points. The graph of the sequence $a_n = \frac{1}{n}$ is similar to $f(x)=\frac{1}{x}$ *except* it only contains the points whose x-coordinates are positive integers.

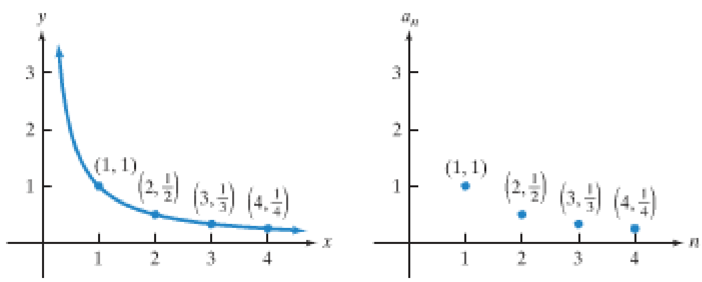

Figure 8.1(a) The graph of $f(x) = \frac{1}{x}, x > 0$

Figure 8.1(b) The graph of $\{a_n\} = \left\{\frac{1}{n}\right\}$

Comparing a continuous graph to the graph of a sequence

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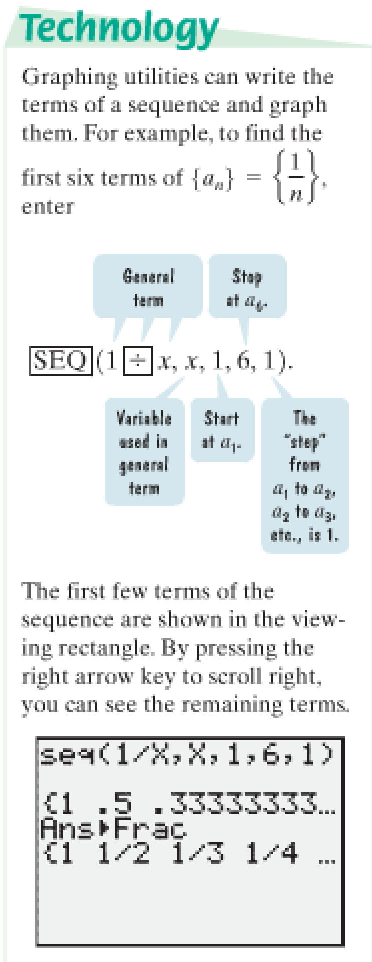
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Graphing utilities can write the terms of a sequence and graph them. For example, to find the first six terms of $\{a_n\} = \left\{\frac{1}{n}\right\}$, enter

The first few terms of the sequence are shown in the viewing rectangle. By pressing the right arrow key to scroll right, you can see the remaining terms.

On the TI 83, 84 Calculator: 2nd - STAT (list) - OPS- #5 for Seq (Expression, X, Lower Bound, Upper Bound, increment)

Example – Write the first four terms of the sequences whose nth term is given below.

(1) $a_n = -2n + 1$

(2) $a_n = \dfrac{(-1)^n}{2^n + 3}$

Recursion Formulas

Sequences can also be defined using recursion formulas. A recursion formula defines the nth term of a sequence as a function of the previous term.

Example – Find the first four terms of a sequence given that $a_1 = 2$ and $a_n = 4(a_{n-1}) + 1$.

Factorial Notation

Factorial Notation

If n is a positive integer, the notation $n!$ (read "n factorial") is the product of all positive integers from n down through 1.

$$n! = n(n-1)(n-2)\cdots(3)(2)(1)$$

0! (zero factorial), by definition, is 1.

$$0! = 1$$

Factorials from 0 to 15

0!	1
1!	1
2!	2
3!	6
4!	24
5!	120
6!	720
7!	5040
8!	40,320
9!	362,880
10!	3,628,800
11!	39,916,800
12!	479,001,600
13!	6,227,020,800
14!	87,178,291,200
15!	1,307,674,368,000

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Most calculators have factorial keys. To find 5!, most calculators use one of the following:

Many Scientific Calculators

5 [x!]

Many Graphing Calculators

5 [!] [ENTER].

Because *n*! becomes quite large as *n* increases, your calculator will display these larger values in scientific notation.

On the TI 83, 84 Calculator: MATH-PRB-#4 is !

Example – Evaluate the factorial expression

$\frac{8!}{2!6!} =$

Example –

Write the first three terms of the sequence whose nth term is $a_n = \frac{3^n}{(n+1)!}$

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Summation Notation

Summation Notation

The sum of the first n terms of a sequence is represented by the **summation notation**

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + a_4 + \cdots + a_n,$$

where i is the **index of summation**, n is the **upper limit of summation**, and 1 is the **lower limit of summation**.

The letter i is called the index of summation and is not related to the use of i represented by $\sqrt{i}$
We often write out a sum that is given in summation notation, we say that we are expanding the summation notation.

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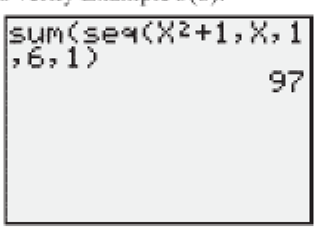
Graphing utilities can calculate the sum of a sequence. For example, to find the sum of the sequence in Example 5(a), enter

SUM SEQ $(x^2 + 1, x, 1, 6, 1)$.

Then press ENTER; 97 should be displayed. Use this capability to verify Example 5(b).



On the TI 83, 84 Calculator: 2nd - STAT (list)-MATH-#5 for Sum
2nd - STAT(list)-OPS-#5 for Seq
Then in parenthese type: (expression, X, lower bound, upper bound, increment)

Table 8.2 Properties of Sums

Property	Example
1. $\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$, c any real number	$\sum_{i=1}^{4} 3i^2 = 3\cdot 1^2 + 3\cdot 2^2 + 3\cdot 3^2 + 3\cdot 4^2$ $3\sum_{i=1}^{4} i^2 = 3(1^2 + 2^2 + 3^2 + 4^2) = 3\cdot 1^2 + 3\cdot 2^2 + 3\cdot 3^2 + 3\cdot 4^2$ Conclusion: $\sum_{i=1}^{4} 3i^2 = 3\sum_{i=1}^{4} i^2$
2. $\sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$	$\sum_{i=1}^{4} (i + i^2) = (1 + 1^2) + (2 + 2^2) + (3 + 3^2) + (4 + 4^2)$ $\sum_{i=1}^{4} i + \sum_{i=1}^{4} i^2 = (1 + 2 + 3 + 4) + (1^2 + 2^2 + 3^2 + 4^2)$ $= (1 + 1^2) + (2 + 2^2) + (3 + 3^2) + (4 + 4^2)$ Conclusion: $\sum_{i=1}^{4} (i + i^2) = \sum_{i=1}^{4} i + \sum_{i=1}^{4} i^2$
3. $\sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$	$\sum_{i=3}^{5} (i^2 - i^3) = (3^2 - 3^3) + (4^2 - 4^3) + (5^2 - 5^3)$ $\sum_{i=3}^{5} i^2 - \sum_{i=3}^{5} i^3 = (3^2 + 4^2 + 5^2) - (3^3 + 4^3 + 5^3)$ $= (3^2 - 3^3) + (4^2 - 4^3) + (5^2 - 5^3)$ Conclusion: $\sum_{i=3}^{5} (i^2 - i^3) = \sum_{i=3}^{5} i^2 - \sum_{i=3}^{5} i^3$

Example – Expand and Evaluate the Sum:

$\sum_{i=1}^{5}(i^2 - 1)$ $\qquad$ $sumseq(x^2 - 1, x, 1,5,1)$

$\sum_{i=2}^{6} i$ $\qquad$ $sumseq(x, x, 2, 6,1)$

Example –

Express each sum using summation notation:

4+6+8+10+12

$3^2 + 4^2 + 5^2 + 6^2 + 7^2$

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27}$$

Example –

Find $\sum_{i=3}^{5}(a_{i-1} + 4)$ where $a_2 = 5$

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